Théorie des goupilles de raquette

Théorie avancée des goupilles de raquette

Comparaison des énergies de déformation

Caractéristiques du spiral avec une spire externe semi-circulaire

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5 \times 10^{-3}\,mm^2$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L_{sp} = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56 \times 10^3\,deg \quad E = 2.093 \times 10^{11}\,m^{-2}N$

Position du piton $\quad r_P := 0.5\cdot d_{piton} \quad \alpha_P := 0 \quad x_P := r_P\cdot cos(\alpha_P) \quad y_P := r_P\cdot sin(\alpha_P)$

$x_P = 2.55\,mm \quad y_P = 0\,mm \quad z_P := x_P + i\cdot y_P$

Position du point d'attache à la virole $\quad r_V := 0.5\cdot d1_{sp} \quad \alpha_V(\theta) := \psi_0 + \theta \quad x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Position du point de raccordement sur le spiral $\quad \alpha_A := 180\cdot deg \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Spire externe formée d'un demi-cercle $\quad R_0 := r_P \quad x_{0t}(\alpha_t) := R_0\cdot cos(\alpha_t) \quad y_{0t}(\alpha_t) := R_0\cdot sin(\alpha_t) \quad z_{0t}(\alpha_t) := R_0\cdot e^{i\cdot\alpha_t}$

$s_t(\alpha_t) := R_0\cdot\alpha_t \quad l_t := s_t(\alpha_A) \quad l_t = 8.011\,mm$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_A - a\cdot(\alpha - \alpha_A) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$s_s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A^{\,2} - r_s(\alpha)^2\right) \quad s_s(\alpha) := r_A\cdot(\alpha - \alpha_A) - \dfrac{a}{2}\cdot(\alpha - \alpha_A)^2 \quad L_t := s_s(\psi_0 + \alpha_A) + l_t$

$L_t = 11.983\,cm$

Position angulaire des goupilles par rapport au piton:

$\varepsilon := 0.06 \quad s_g := \varepsilon\cdot L_t \quad s_g = 7.19\,mm \quad \alpha_g := \dfrac{s_g}{R_0} \quad \alpha_g = 161.548\,deg$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Position radiale de la goupille pour une élongation de contact donnée

Première approximation de la déformée de la spire externe entre piton et goupille $\quad \theta_1 := 20\cdot deg$

$x_{0g}(\alpha_g) := R_0\cdot cos(\alpha_g) \quad y_{0g}(\alpha_g) := R_0\cdot sin(\alpha_g)$

$\xi_{0g}(\alpha_g) := \dfrac{R_0}{\alpha_g}\cdot sin(\alpha_g) \quad \eta_{0g}(\alpha_g) := \dfrac{R_0}{\alpha_g}\cdot(1 - cos(\alpha_g))$

$$\beta := arctan\left[\frac{-(x_{0g}(\alpha_g) - \xi_{0g}(\alpha_g))}{y_{0g}(\alpha_g) - \eta_{0g}(\alpha_g)}\right] + \pi$$

Jeu entre spiral et goupille au repos $\quad j := \dfrac{\varepsilon\cdot(y_{0g}(\alpha_g) - \eta_{0g}(\alpha_g))}{cos(\beta)}\cdot\theta_1 \quad j = 0.06\,mm$

Calcul de la matrice $\boldsymbol{D_g}$

Calcul à partir de la première approximation de la déformée

$$z_{1t}(\theta,\alpha_t) := z_P + \frac{L_t \cdot R_0}{L_t + \theta \cdot R_0} \cdot \left(\exp\left(i \cdot \alpha_t \cdot \frac{L_t + \theta \cdot R_0}{L_t} \right) - 1 \right)$$

$x_1(\theta,\alpha) := Re(z_{1t}(\theta,\alpha))$ $\quad$ $y_1(\theta,\alpha) := Im(z_{1t}(\theta,\alpha))$ $\quad$ $EI := E \cdot I_s \cdot N^{-1} \cdot m^{-2}$ $\quad$ $EI = 7.063 \times 10^{-8}$

$$\xi_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} x_1(\theta_0,\alpha)\, d\alpha \qquad \eta_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} y_1(\theta_0,\alpha)\, d\alpha$$

$$p2_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} x_1(\theta_0,\alpha)^2\, d\alpha \qquad q2_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} y_1(\theta_0,\alpha)^2\, d\alpha$$

$$k_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} x_1(\theta_0,\alpha) \cdot y_1(\theta_0,\alpha)\, d\alpha$$

$d1_{11}(\alpha_g) := q2_{1g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d1_{22}(\alpha_g) := p2_{1g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d1_{33}(\alpha_g) := 1$ $\quad$ $\mathrm{R}_0 := R_0 \cdot m^{-1}$

$d1_{12}(\alpha_g) := -k_{1g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d1_{13}(\alpha_g) := \eta_{1g}(\alpha_g) \cdot m^{-1}$ $\quad$ $d1_{23}(\alpha_g) := -\xi_{1g}(\alpha_g) \cdot m^{-1}$

$$\boldsymbol{D_{1g}}(\alpha_g) := \frac{\mathrm{R}_0 \cdot \alpha_g}{EI} \cdot \begin{pmatrix} d1_{11}(\alpha_g) & d1_{12}(\alpha_g) & d1_{13}(\alpha_g) \\ d1_{12}(\alpha_g) & d1_{22}(\alpha_g) & d1_{23}(\alpha_g) \\ d1_{13}(\alpha_g) & d1_{23}(\alpha_g) & 1 \end{pmatrix} \qquad \boldsymbol{D_{1g}}(\alpha_g) = \begin{pmatrix} 0.277 & -0.035 & 152.033 \\ -0.035 & 0.277 & -26.646 \\ 152.033 & -26.646 & 1.018 \times 10^5 \end{pmatrix}$$

Approximation à partir de la forme naturelle du spiral

$$p2_{0g}(\alpha_g) := \frac{R_0^2}{2 \cdot \alpha_g} \cdot \left(\alpha_g + \frac{\sin(2 \cdot \alpha_g)}{2} \right) \qquad q2_{0g}(\alpha_g) := \frac{R_0^2}{2 \cdot \alpha_g} \cdot \left(\alpha_g - \frac{\sin(2 \cdot \alpha_g)}{2} \right) \qquad k_{0g}(\alpha_g) := \frac{R_0^2}{2 \cdot \alpha_g} \cdot \sin(\alpha_g)^2$$

$d_{11}(\alpha_g) := q2_{0g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d_{22}(\alpha_g) := p2_{0g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d_{33}(\alpha_g) := 1$

$d_{12}(\alpha_g) := -k_{0g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d_{13}(\alpha_g) := \eta_{0g}(\alpha_g) \cdot m^{-1}$ $\quad$ $d_{23}(\alpha_g) := -\xi_{0g}(\alpha_g) \cdot m^{-1}$

$$\boldsymbol{D_g}(\alpha_g) := \frac{\mathrm{R}_0 \cdot \alpha_g}{EI} \cdot \begin{pmatrix} d_{11}(\alpha_g) & d_{12}(\alpha_g) & d_{13}(\alpha_g) \\ d_{12}(\alpha_g) & d_{22}(\alpha_g) & d_{23}(\alpha_g) \\ d_{13}(\alpha_g) & d_{23}(\alpha_g) & 1 \end{pmatrix} \qquad \boldsymbol{D_g}(\alpha_g) = \begin{pmatrix} 0.366 & -0.012 & 179.392 \\ -0.012 & 0.296 & -29.138 \\ 179.392 & -29.138 & 1.018 \times 10^5 \end{pmatrix}$$

Calcul des formes quadratiques

$Delta(\alpha_g) := \sin(\alpha_g)$ $\quad$ $\gamma(\alpha_g) := \cos(\alpha_g)$ $\quad$ $x_{0g}(\alpha_g) := R_0 \cdot \cos(\alpha_g)$ $\quad$ $y_{0g}(\alpha_g) := R_0 \cdot \sin(\alpha_g)$

$$H(\alpha_g) := \frac{R_0}{\alpha_g} \cdot (1 - \cos(\alpha_g)) \qquad J(\alpha_g) := \frac{R_0}{\alpha_g} \cdot (\alpha_g - \sin(\alpha_g))$$

$$\boldsymbol{V_1}(\alpha_g) := \begin{pmatrix} \gamma(\alpha_g) & Delta(\alpha_g) & 0 \end{pmatrix}^T \qquad \boldsymbol{V_2}(\alpha_g) := \begin{pmatrix} Delta(\alpha_g) & -\gamma(\alpha_g) & -\mathrm{R}_0 \end{pmatrix}^T$$

$$W1_{g1}(\alpha_g) := \frac{1 \cdot s^2 \cdot kg^{-1}}{2} \cdot \mathbf{V_1}(\alpha_g)^T \cdot \mathbf{D_{1g}}(\alpha_g) \cdot \mathbf{V_1}(\alpha_g) \qquad W1_{g1}(\alpha_g) = 0.149\ kg^{-1} \cdot s^2$$

$$W_{g1}(\alpha_g) := \frac{R_0^{\,3}}{4 \cdot E \cdot I_s} \cdot (\alpha_g - \sin(\alpha_g) \cdot \cos(\alpha_g)) \qquad W_{g1}(\alpha_g) = 0.183\ kg^{-1} \cdot s^2$$

$$W1_{g2}(\alpha_g) := \frac{1 \cdot s^2 \cdot kg^{-1}}{2} \cdot \mathbf{V_2}(\alpha_g)^T \cdot \mathbf{D_{1g}}(\alpha_g) \cdot \mathbf{V_2}(\alpha_g) \qquad W1_{g2}(\alpha_g) = 0.4\ kg^{-1} \cdot s^2$$

$$W_{g2}(\alpha_g) := \frac{R_0^{\,3}}{4 \cdot E \cdot I_s} \cdot \left[3 \cdot \alpha_g + \sin(\alpha_g) \cdot (\cos(\alpha_g) - 4)\right] \qquad W_{g2}(\alpha_g) = 0.405\ kg^{-1} \cdot s^2$$

$$W1_{g3}(\alpha_g) := \frac{1 \cdot s^2 \cdot kg^{-1}}{2} \cdot \mathbf{V_1}(\alpha_g)^T \cdot \mathbf{D_{1g}}(\alpha_g) \cdot \mathbf{V_2}(\alpha_g) \qquad W1_{g3}(\alpha_g) = 0.209\ kg^{-1} \cdot s^2$$

$$W_{g3}(\alpha_g) := \frac{R_0^{\,3}}{4 \cdot E \cdot I_s} \cdot (1 - \cos(\alpha_g))^2 \qquad W_{g3}(\alpha_g) = 0.223\ kg^{-1} \cdot s^2$$

$$V_g(\alpha_g) := \begin{pmatrix} R_0 \cdot \sin(\alpha_g) & -R_0 \cdot \cos(\alpha_g) & 1 \end{pmatrix}^T$$

$$W1_{c2}(\alpha_g) := \frac{1 \cdot kg \cdot m^2 \cdot s^{-2}}{2} \cdot \left(\left(V_g(\alpha_g)\right)\right)^T \cdot \mathbf{D_{1g}}(\alpha_g)^{-1} \cdot V_g(\alpha_g) \qquad W1_{c2}(\alpha_g) = 2.358 \times 10^{-5}\ m^2 \cdot kg \cdot s^{-2}$$

$$\Delta_g(\alpha_g) := \frac{R_0^{\,4}}{4 \cdot \alpha_g^{\,3}} \cdot (\alpha_g - \sin(\alpha_g)) \cdot \left[\alpha_g \cdot (\alpha_g + \sin(\alpha_g)) - 4 \cdot (1 - \cos(\alpha_g))\right] \qquad \Delta_g(\alpha_g) = 1.237\,mm^4$$

$$W_{c2}(\alpha_g) := \frac{E \cdot I_s \cdot R_0^{\,3}}{8 \cdot \Delta_g(\alpha_g) \cdot \alpha_g^{\,3}} \cdot \left[3 \cdot \alpha_g^{\,2} - 2 \cdot \alpha_g \cdot \sin(\alpha_g) \cdot (2 + \cos(\alpha_g)) - (1 - \cos(\alpha_g)) \cdot (1 - 7 \cdot \cos(\alpha_g))\right]$$

$$W_{c2}(\alpha_g) = 3.741 \times 10^{-5}\ m^2 \cdot kg \cdot s^{-2}$$

Réaction normale à la goupille

Coefficient de frottement

$$\mu_g := 0.1$$

$$F1_{gn}(\theta, \alpha_g) := \frac{\varepsilon \cdot H(\alpha_g) \cdot (\theta - \theta_1)}{2 \cdot (W1_{g1}(\alpha_g) - \mu_g \cdot W1_{g3}(\alpha_g))} \qquad F1_{gn}(\theta_0, \alpha_g) = 1.8 \times 10^{-3}\ N$$

$$F_{gn}(\theta, \alpha_g) := \varepsilon \cdot (\theta - \theta_1) \cdot \frac{2 \cdot E \cdot I_s}{R_0^{\,2}} \cdot \frac{1 - \cos(\alpha_g)}{\alpha_g \cdot \left[\alpha_g - \sin(\alpha_g) \cdot \cos(\alpha_g) - \mu_g \cdot (1 - \cos(\alpha_g))^2\right]}$$

$$F_{gn}(\theta_0, \alpha_g) = 1.434 \times 10^{-3}\ N$$

Accroissement de l'angle polaire de la tangente au spiral au point de contact

$$N1(\alpha_g) := \frac{E \cdot I_s}{2 \cdot R_0 \cdot \alpha_g \cdot W1_{c2}(\alpha_g)} \qquad N1(\alpha_g) = 0.208$$

$$N(\alpha_g) := \frac{E \cdot I_s}{2 \cdot R_0 \cdot \alpha_g \cdot W_{c2}(\alpha_g)} \qquad N(\alpha_g) = 0.131 \qquad \varphi_{sg}(\theta, \alpha_g) := \varepsilon \cdot (\theta - \theta_1) \cdot (N(\alpha_g) - 1)$$

$\Delta\varphi_g := \varepsilon\cdot(\theta_0 - \theta_1)\cdot N(\alpha_g)$ $\Delta\varphi_g = 1.969\,deg$ $\Delta\varphi_g := \varepsilon\cdot(\theta_0 - \theta_1)\cdot N1(\alpha_g)$ $\Delta\varphi_g = 3.125\,deg$

Sans goupille $\Delta\varphi := \theta_0\cdot\dfrac{R_0\cdot\alpha_g}{L_t}$ $\Delta\varphi = 16.2\,deg$

Comparaison des ordres de grandeurs des énergies de déformation

Par le fichier de simulation "Spiral avec courbe terminale concentrique" on obtient, en première approximation, les réactions suivantes

- Cas sans goupille $\alpha_A = 180\,deg$ $R'_x := -1.09\cdot 10^{-4}\cdot N$ $R'_y := -2.62\cdot 10^{-5}\cdot N$

- Cas d'un encastrement aux goupilles $\alpha_A - \alpha_g = 18.452\,deg$ $R_x := -8.51\cdot 10^{-5}\cdot N$ $R_y := 6.45\cdot 10^{-6}\cdot N$

On calculera les éléments des matrice ***D*** à partir de la forme naturelle du spiral

Energie de déformation totale sans goupilles

$$\xi_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} x_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} x_{0t}(\alpha_t)\cdot R_0\,d\alpha_t\right) \qquad \xi_{0s} = 1.27\times 10^{-3}\,mm$$

$$\eta_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} y_{0t}(\alpha_t)\cdot R_0\,d\alpha_t\right) \qquad \eta_{0s} = 0.064\,mm$$

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} x_{0t}(\alpha_t)^2\cdot R_0\,d\alpha_t\right) \qquad p2_{0s} = 1.361\,mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} y_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} y_{0t}(\alpha_t)^2\cdot R_0\,d\alpha_t\right) \qquad q2_{0s} = 1.598\,mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} x_{0t}(\alpha_t)\cdot y_{0t}(\alpha_t)\cdot R_0\,d\alpha_t\right) \qquad k_{0s} = 0.186\,mm^2$$

$$W_1 := \frac{L_t}{2\cdot E\cdot I_s}\cdot\left(q2_{0s}\cdot R'_x{}^2 + p2_{0s}\cdot R'_y{}^2 - 2\cdot k_{0s}\cdot R'_x\cdot R'_y\right) \qquad W_1 = 1.599\times 10^{-8}\,m\,N$$

Energie de déformation totale avec les goupilles comme encastrement

$$W_v := \frac{L_t}{2\cdot E\cdot I_s}\cdot\left(q2_{0s}\cdot R_x{}^2 + p2_{0s}\cdot R_y{}^2 - 2\cdot k_{0s}\cdot R_x\cdot R_y\right) \qquad W_v = 1.004\times 10^{-8}\,m\,N$$

Variation d'énergie de déformation totale due à la présence de goupille

$$\Delta R_x := R_x - R'_x \qquad \Delta R_y := R_y - R'_y \qquad \Delta C_z := -\frac{E\cdot I_s}{L_t}\cdot\varphi_{sg}(\theta_0,\alpha_g)$$

$$\Delta W_v := \frac{L_t}{2\cdot E\cdot I_s}\cdot\left(q2_{0s}\cdot\Delta R_x{}^2 + p2_{0s}\cdot\Delta R_y{}^2 + \Delta C_z{}^2 - 2\cdot k_{0s}\cdot\Delta R_x\cdot\Delta R_y + 2\cdot\eta_{0s}\cdot\Delta R_x\cdot\Delta C_z - 2\cdot\xi_{0s}\cdot\Delta R_y\cdot\Delta C_z\right)$$

$$\Delta W_v = 1.734\times 10^{-8}\,m\,N$$

Energie de déformation due aux réactions sur les goupilles

$$F_{gt}(\theta, \alpha_g) := \mu_g \cdot F_{gn}(\theta, \alpha_g) \qquad A := \begin{pmatrix} \gamma(\alpha_g) & Delta(\alpha_g) \\ -Delta(\alpha_g) & \gamma(\alpha_g) \end{pmatrix} \qquad \mathbf{F_g} := A^{-1} \cdot \begin{pmatrix} F_{gn}(\theta_0, \alpha_g) \\ F_{gt}(\theta_0, \alpha_g) \end{pmatrix}$$

$$\mathbf{F_g} = \begin{pmatrix} -1.406 \times 10^{-3} \\ 3.179 \times 10^{-4} \end{pmatrix} N \qquad \Gamma_g := -y_{0g}(\alpha_g) \cdot \mathbf{F_g}_0 + x_{0g}(\alpha_g) \cdot \mathbf{F_g}_1 \qquad \Gamma_g = 3.658 \times 10^{-7}\ m N$$

$$\mathbf{T_g} := \left[\mathbf{F_g}_0 \cdot N^{-1} \quad \mathbf{F_g}_1 \cdot N^{-1} \quad \Gamma_g \cdot (N \cdot m)^{-1} \right]^T \qquad W_g := \frac{1 \cdot N \cdot m}{2} \cdot \mathbf{T_g}^T \cdot \mathbf{D_g}(\alpha_g) \cdot \mathbf{T_g} \qquad W_g = 2.934 \times 10^{-7}\ m N$$

Energie de déformation sur la portion piton - goupille due aux réactions à la virole

$$\mathbf{T_v} := \left(R_x \cdot N^{-1} \quad R_y \cdot N^{-1} \quad 0 \right)^T \qquad W_{vg} := \frac{1 \cdot N \cdot m}{2} \cdot \mathbf{T_v}^T \cdot \mathbf{D_g}(\alpha_g) \cdot \mathbf{T_v} \qquad W_{vg} = 1.339 \times 10^{-9}\ m N$$